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Relation Between the Dark Energy Density and Temperature

H. Saadat

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Abstract In this article we investigate the relation between the temperature and density of the dark energy. We find that the temperature of the dark universe is proportional to the inverse of dark energy density. Also we discuss some values of the important parameters of the theory.

Keywords Dark energy · Cosmological constant · Thermodynamics

1 Introduction

It is found that a large amount of the energy in our universe is of unknown nature to us which called dark energy. According to the latest studies and astronomical observations such as WMAP (Wilkinson microwave anisotropy probe [1]) our universe consists of about 27% matter and 73% dark energy. Nowadays it is clear that our universe expand with acceleration [2–6]. This accelerating expansion of the universe may be explained in context of the dark energy, because of negative pressure, and the simplest model for the dark energy is the Einstein's cosmological constant. On the other hand the study of the cosmological constant is one of the important problems in theoretical and experimental physics [7–11]. Another candidate for the dark energy is scalar-field dark energy model [12–29]. However there are many unsuccessful efforts to specify the nature of the dark energy. The reason is that the dark energy problem fallen in to an issue of quantum gravity which is unknown theory today. Just there is a nice candidate for the quantum gravity which is the string theory. Recent study [30] suggests that there exist a large number of consistent in string theory [31]. Recently, the weak gravity conjecture has been applied to explain the dark energy which called holographic dark energy model [32–37]. One of the important ways to specify the nature of the dark energy is the study of the time-dependent dark energy density. For the

H. Saadat (✉)
Science Faculty, Department of Physics, Islamic Azad University, Marvdasht, Iran,
P.O. Box 71555-477, Shiraz, Iran
e-mail: hsaadat2002@yahoo.com

similar reason the relation between dark matter density and temperature of the dark universe have already been studied [38]. Also the effect of dark matter on the solar system have already considered [39, 40] which can be extended to the case of the dark energy. In this paper we wish to investigate the relation between the dark energy density and temperature of the dark universe. In that case we can fix parameters of this theory in several ways. It is found the differential equation for time-dependent dark energy density and temperature as the following,

$$\frac{dT}{dt}+\frac{T}{\rho}\frac{d\rho}{dt}+\frac{\gamma}{t^2}=0, \tag{1}$$

where $\gamma=(1-2/3\beta_{de})\frac{E}{8\pi}$ with β_{de} is the only free parameter in dark energy anisotropy, which can be treated as a constant for simplicity. The constant E has energy dimension, also we assumed that $8\pi G=1$.

2 Dark Energy Density

The time-dependent dark energy density may be given by,

$$\rho=\rho(0)e^{\alpha t}(1+t)^{\beta}, \tag{2}$$

where $\rho(0)$ is the density of dark energy at the initial time, α and β are constant parameters which we will fix them by using equation of state in the next section. Substituting the dark energy density into (1) yields us to the following differential equation,

$$\frac{dT}{dt}+T\left(\alpha+\frac{\beta}{1+t}\right)+\frac{\gamma}{t^2}=0. \tag{3}$$

General solution of this equation may be written as,

$$T=e^{-\alpha t}(1+t)^{-\beta}\left[C-\int\frac{\gamma e^{\alpha t}(1+t)^{\beta}}{t^2}dt\right], \tag{4}$$

where C is the integration constant. Our major goal in this paper is to solve above equation and obtain explicit expression for the temperature. First of all we look at the homogeneous part of the differential equation (3), which obtained by taking $\gamma=0$. In that case one can obtain,

$$T=Ce^{-\alpha t}(1+t)^{-\beta}, \tag{5}$$

which implies that $T\rho=C\rho(0)$. If we parameterize our solution as $T\rho=1$ then one can find $C=\rho(0)^{-1}=\frac{L^2}{3}$ where L is the spatial size of the universe. Now we use asymptotic expansion of the relation (4) and find,

$$T=\gamma\left[\frac{1}{t}-(\alpha+\beta)(1+\ln t)\right]+\frac{L^2}{3}. \tag{6}$$

For the $t\to\infty$ limit we can obtain,

$$T=-\gamma(\alpha+\beta)\ln t. \tag{7}$$

Now we try to obtain explicit expression of the temperature for different values of β. For the $\beta=1$ one can obtain,

$$T = e^{-\alpha t}\frac{\gamma e^{\alpha t} + \gamma\alpha t Ei(1, -\alpha t) + \gamma t Ei(1, -\alpha t) + \frac{L^2}{3}t}{t(1+t)}, \tag{8}$$

where,

$$Ei(1, -\alpha t) = \int \frac{e^{\alpha t x}}{x} dx. \tag{9}$$

For the $\beta = 2$ one can obtain,

$$T = e^{-\alpha t}\frac{\gamma e^{\alpha t} + \gamma\alpha^2 t Ei(1, -\alpha t) + 2\gamma\alpha t Ei(1, -\alpha t) - \gamma t e^{\alpha t} + \frac{\alpha L^2}{3}t}{\alpha t(1+t)^2}. \tag{10}$$

One can continue this procedure and find general expression for the temperature as the following,

$$T = e^{-\alpha t}(1+t)^{-\beta}\left[\frac{L^2}{3} + \frac{\gamma}{t}e^{\alpha t} + \gamma(\alpha+\beta)Ei(1, -\alpha t)\right]. \tag{11}$$

By using the asymptotic expansion of the $Ei(1, -\alpha t)$ the relation (11) reduced to the following expression,

$$T = e^{-\alpha t}(1+t)^{-\beta}\left[\frac{L^2}{3} + \frac{\gamma}{t}e^{\alpha t} - \gamma^2(\alpha+\beta) - \gamma(\alpha+\beta)\ln t - \alpha\gamma(\alpha+\beta)t + \mathcal{O}(t^2)\right] \tag{12}$$

which agree with (7) at the $t \to \infty$ limit. In the next section we consider (11) and equation of state for the dark energy and try to fix corresponding parameters.

3 Equation of State

Here we consider simple equation of state,

$$\omega(t) = \omega_0 + \omega_1 t, \tag{13}$$

where, in analogues to the previous section, $\alpha = 3\omega_1$ and $\beta = 3(1 + \omega_0 - \omega_1)$. In this paper we set $E = 8\pi$ and $\beta_{de} = 1$ hence $\gamma = 1/3$. For the simplest case of constant dark energy density one can choose $\omega_0 = -1$ and $\omega_1 = 0$, therefore $\rho = \frac{3}{L^2}$ and $T = \frac{1}{3}t + \rho^{-1}$ so the temperature of the dark universe decreases with time to the constant dark energy density. In that case the early universe was very hot with very high density. This situation is known as the cosmological constant model of the dark energy.

Another interesting model of the dark energy is a quintessence model where $\omega_0 = -1$ and $\omega_1 = 0.5$, therefore,

$$\rho = \frac{3}{L^2}\frac{e^{\frac{3}{2}t}}{(1+t)^{\frac{3}{2}}},$$

$$T = (1+t)^{\frac{3}{2}}e^{-\frac{3}{2}t}\left[\frac{L^2}{3} + \frac{e^{\frac{3}{2}t}}{3t}\right]. \tag{14}$$

In Fig. 1a we plot the temperature in terms of time for the early universe and show that the temperature decreases suddenly. Then the temperature grows up for the late time. We plot this part of the temperature in Fig. 1b.

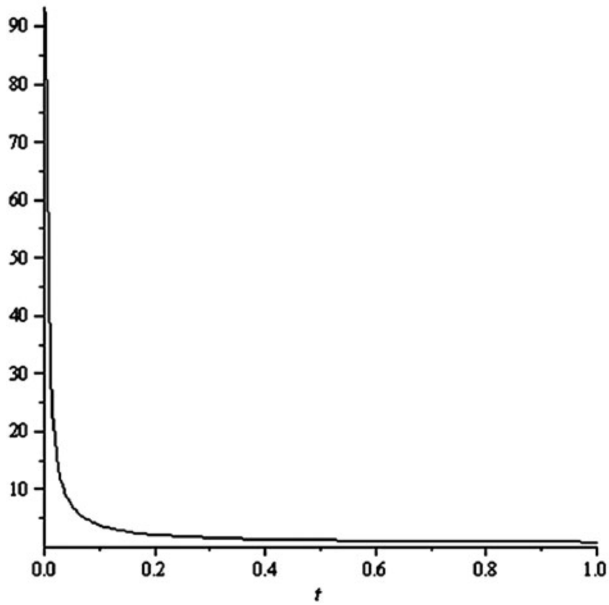


Fig. 1a Temperature versus time for the early universe

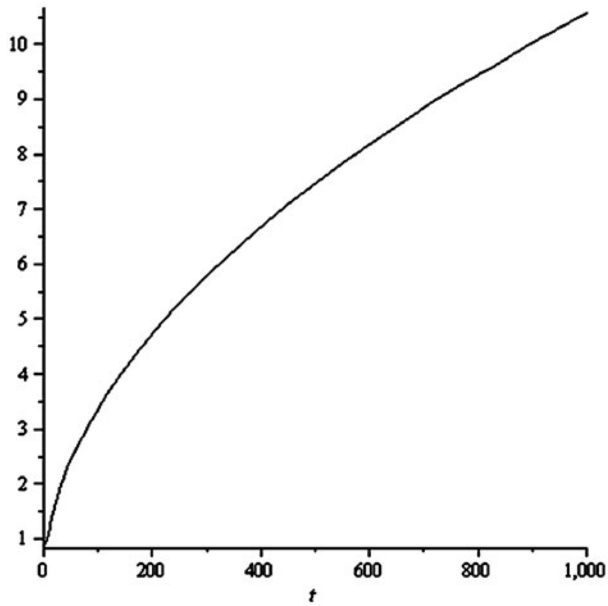


Fig. 1b Temperature versus time for the late time

Also there is another model where one can choose $\omega_0 = \omega_1 = -1$ so one can obtain,

$$\rho = \frac{3}{L^2} e^{-3t} (1+t)^3,$$
$$T = (1+t)^{-3} e^{3t} \left[\frac{L^2}{3} + \frac{e^{-3t}}{3t} \right]. \tag{15}$$

Fig. 2a Temperature versus time for the early universe

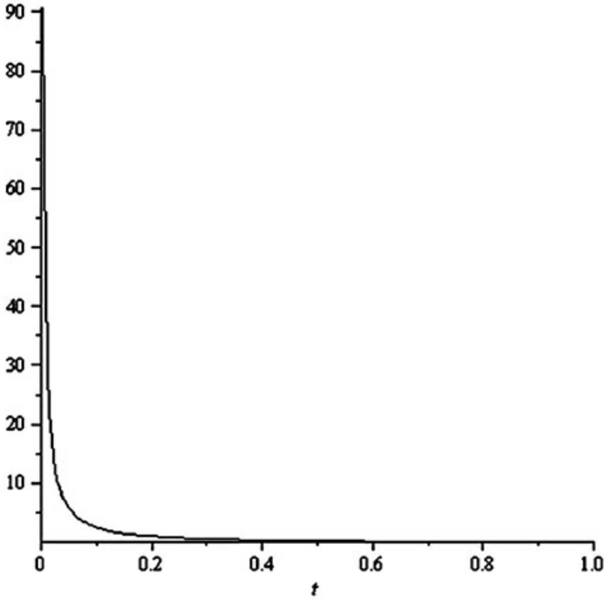
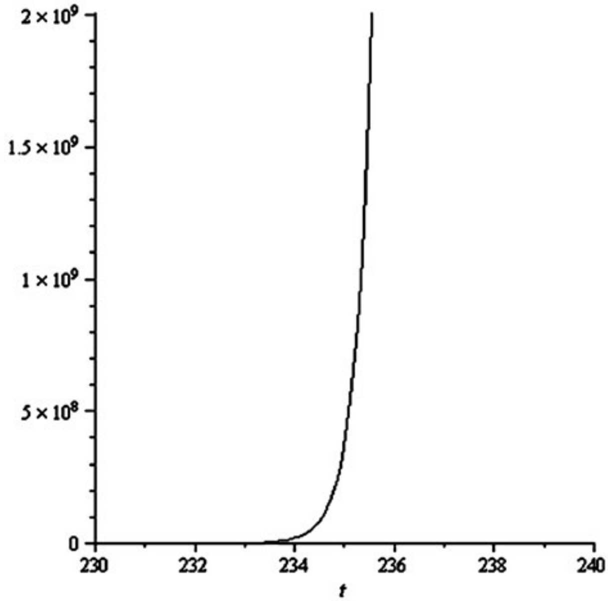


Fig. 2b Temperature versus time for the late time



It is clear that the dark energy density of (14) is increasing but (15) is decreasing. However the temperature for both cases are decreasing in the early universe and increasing in the late time. We plot the temperature in terms of time for the early universe in Fig. 2a and show that the temperature decreases suddenly. Then in Fig. 2b we plot the temperature in terms of late time and show that the temperature grow up suddenly.

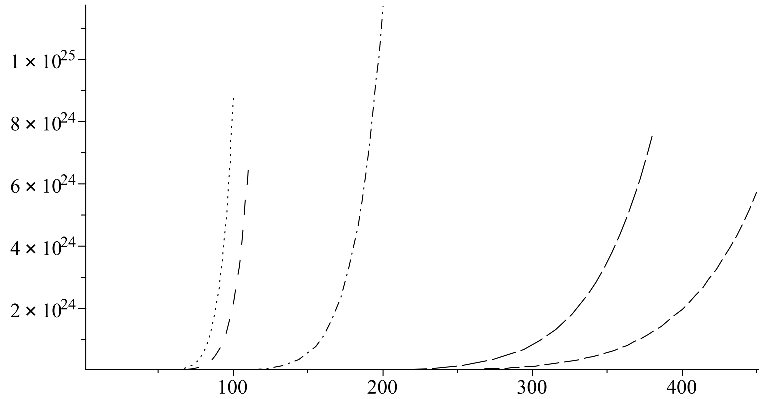
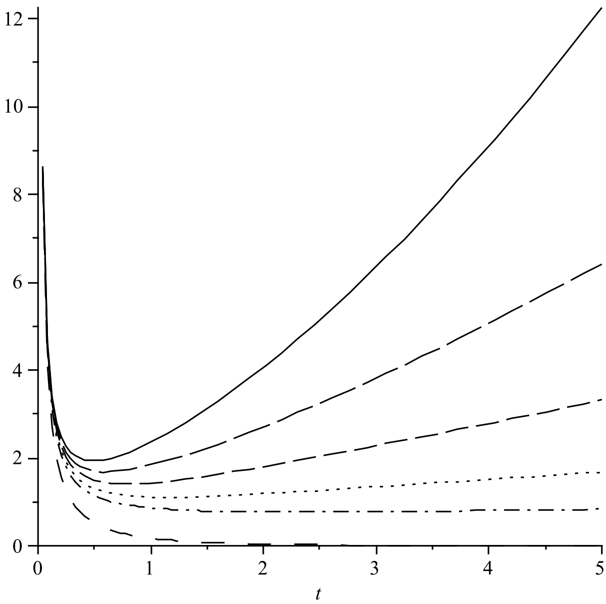


Fig. 3a Temperature versus time for $-5 \le \omega_0 \le -4$. The *dotted line* denotes $\omega_0 = -5$, *space dashed line* denotes $\omega_0 = -4.9$, *dash dotted line* denotes $\omega_0 = -4.5$, *long dashed line* denotes $\omega_0 = -4.1$ and *dashed line* denotes $\omega_0 = -4$

Fig. 3b Temperature versus time for $-1.5 \le \omega_0 \le -0.5$. The *solid line* denotes $\omega_0 = -1.5$, *dotted line* denotes $\omega_0 = -1.2$, *space dashed line* denotes $\omega_0 = -0.5$, *dash dotted line* denotes $\omega_0 = -1.1$, *long dashed line* denotes $\omega_0 = -1.4$ and *dashed line* denotes $\omega_0 = -1.3$



Now we consider our case with $\omega_1 = 0$ which yields to the following expressions for the dark energy density and temperature,

$$\rho = \frac{3}{L^2}(1+t)^{3(1+\omega_0)},$$
$$T = (1+t)^{-3(1+\omega_0)}\left[\frac{L^2}{3} - \frac{1+\omega_0}{3} + \frac{1}{3t} - (1+\omega_0)\ln t\right]. \tag{16}$$

In Figs. 3a–3d we plot the temperature in terms of time for various values of ω_0. Already the case of $-1.2 \le \omega_0 \le -0.5$ discussed in the literature. We show that (in Figs. 3a, 3b and 3c)

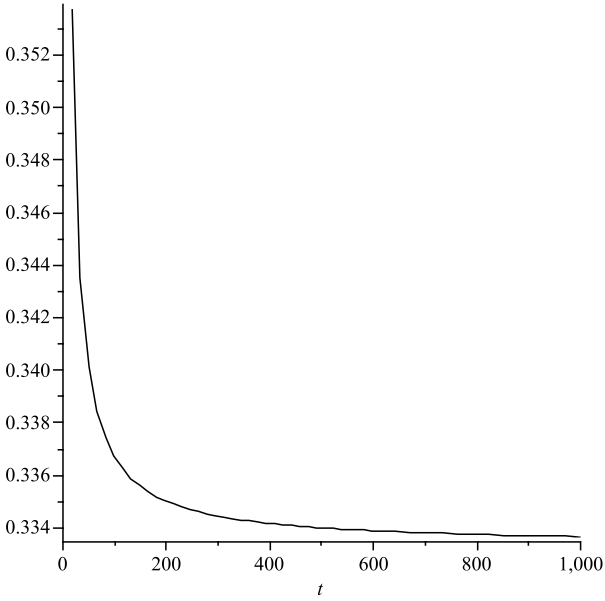


Fig. 3c Temperature versus time for $\omega_0 = -1$

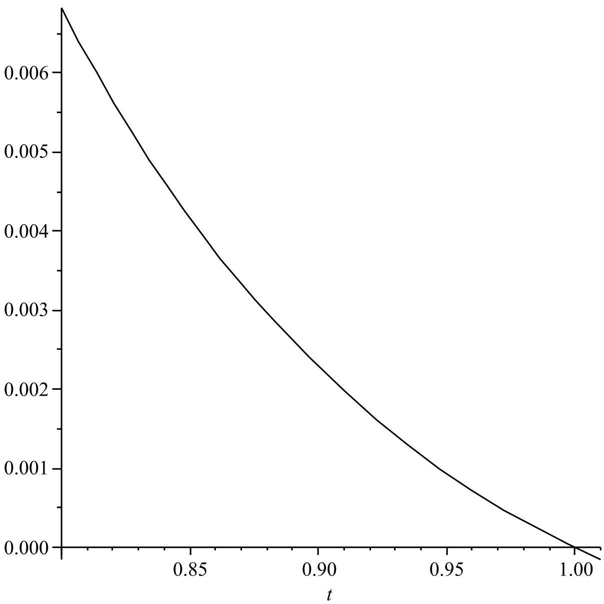


Fig. 3d Temperature versus time for $\omega_0 = 1$. For $t > 1$ the temperature moves to negative region which is not allowed

the temperature decreased in the early universe suddenly and then increased with time. Here we consider possibility of the positive ω_0 and find that for the early time the positive values of ω_0 allowed, then we find time of phase transition of the positive ω_0 to the negative ω_0. We find phase transition of $\omega_0 = 0$ at $t = 1.75$, $\omega_0 = 1$ at $t = 1$, and $\omega_0 = 2$ at $t = 0.785$. In Fig. 3d we plot the case of $\omega_0 = 1$.

4 Conclusion

In this paper we considered time-dependent dark energy density and relation with the temperature. We solved the differential equation and obtained time-dependent temperature of the dark universe. We found the temperature proportional to the inverse of the dark energy density. We found that the temperature for the late time from the big bang as $T = -\frac{1}{3}(\alpha+\beta)\ln t$, then for the case of $\omega_1 = 0$, hence $\alpha = 0$, reduced to $T = -\frac{1}{3}(1+\omega_0)\ln t$. It tell us that for the late time $\omega_0 < -1$ allowed. However we showed that the positive ω_0 may be allowed in the early universe. Also we examine several values of ω_0 and ω_1 corresponding to the cosmological constant and quintessence models.

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